Discussion Point

In Part a, we are simulating the accumulation of a "security score" for M systems subject to N attacks, where the score is -1 if the system is penetrated and 1 if the system is successfully shielded. In Part b, we extend this to simulate the cumulated frequency of penetration (f), relative frequency (f/number of attacks), and the normalized ratio (f/√number of attacks). The expectations from these simulations are:

* Averages of the Distributions: In Part a, we expect the average security score trajectories for the systems to, on average, approach p for each system over time. This means that, on average, we would expect a proportion of p successful shielding and 1-p penetrations for each attack.
* Shapes of the Histograms: In Part a, we expect the histograms of the security score trajectories to show a distribution centered around the expected average value, which is p for a successful shielding and (1-p) for penetration.
* Regularities: As N increases, we should see regularities emerging in both Part a and Part b. With a larger number of attacks (N), we expect the scores to converge to their expected values (p for shielding, 1-p for penetration) and exhibit more regular patterns. In Part b, the cumulated frequency (f) should approach p \* N, and the relative frequency (f/number of attacks) should approach p. As N becomes very large, these values should become increasingly stable and regular.
* Theoretical Limit Distributions: The theoretical limit distributions will depend on the value of p (penetration probability), and they are likely to approach specific values. For Part a, the limit distribution should approach a Bernoulli distribution with parameter p, and for Part b, we can expect a distribution for f to approximate a binomial distribution with parameters N and p.

Exercise 2

A probability space, also known as a probability triple, consists of three fundamental elements:

1. Sample Space (Ω): The sample space is the set of all possible outcomes of a random experiment or process. Each element in the sample space represents a distinct outcome of the experiment.
2. Events (F): Events are subsets of the sample space, representing specific outcomes or sets of outcomes that we are interested in. Events can be simple (a single outcome) or compound (multiple outcomes).
3. Probability Measure (P): The probability measure assigns a probability to each event in the sample space, indicating the likelihood of that event occurring. The probability measure is a function P(F) that maps events to real numbers in the range [0, 1] and satisfies certain axioms of probability. The probability measure P represents the probability distribution over events.

In the context of modeling the Exercise 1, the three sets of the probability space are:

1. Sample Space (Ω): The sample space Ω represents all possible sequences of system states (penetrated or shielded) for M systems over N attacks.
2. Events (F): Events in this context represent specific security score trajectories, such as "at least one system successfully shielded" or "all systems penetrated." Each event corresponds to a particular combination of system states over the N attacks.
3. Probability Measure (P): Probability Measure (P): The probability measure P assigns probabilities to events based on the underlying probability distribution. For instance, P(A) could represent the probability that the event "at least one system is successfully shielded" occurs over N attacks, given the values of M and p.